An introduction to Regge Field Theory

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Map of High Energy Physics

\[
\ln \frac{1}{x} \quad \ln Q^2
\]
The *Scattering Matrix*

The transition of a closed system of particles from an initial state $|k\rangle$ to a final state $|f\rangle$ is described in quantum theory by the $S$ matrix:

$$|f\rangle = S |k\rangle$$

The matrix elements of the $S$ matrix:

$$S_{fk} = \langle f | S | k \rangle$$

Can be represented in the form

$$S_{fk} = \delta_{fk} + i (2\pi)^4 \delta^{(4)}(P_i - P_k) T_{fk}$$

$\delta_{fk} = 1$ if the state does not change ($|f\rangle = |k\rangle$). No interaction.

$T_{fk}$ is called the transition (scattering) amplitude from the state $|k\rangle$ to the state $|f\rangle$.

$T_{fk}$ is a function of 4-momentum and polarization of particles (and contains $\gamma$-matrixes in case of fermions).
The Scattering Amplitude

For spinless particles, $T_{fk}$ is a function of the relativistically invariant variables formed from the 4-momentum of the particles.

$T_{fk}(P_1, P_2, P_3, P_4)$

$P_i = \{E_i, \mathbf{p}_i\}$

$4 \times 4 = 16$ variables

Not all 16 variables are independent

Energy-momentum conservation: $P_4 = P_1 + P_2 - P_3$

6 invariants can be formed with $P_1, P_2$ and $P_3$: $P_1^2, P_2^2, P_3^2, (P_1P_2), (P_1P_3), (P_2P_3)$

$P_i^2 = m_i^2$ ($i = 1, 2, 3, 4$)

$P_4^2 = (P_1 + P_2 - P_3)^2 = P_1^2 + P_2^2 + P_3^2 + 2(P_1P_2) - 2(P_1P_3) - 2(P_2P_3) = m_4^2$

$T_{fk}$ is a function of 2 variables for binary reactions with spinless particles

Mandelstam variables

$s = (P_1 + P_2)^2 = (P_3 + P_4)^2$

t $= (P_1 - P_3)^2 = (P_2 - P_4)^2$ [1+3 \rightarrow 2+4 ]

$u = (P_1 - P_4)^2 = (P_2 - P_3)^2$ [1+4 \rightarrow 2+3 ]

$s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2$

$T_{fk} = T_{fk}(s, t)$
**Crossing symmetry**

In quantum field theory, absorption of a particle with 4-momentum $-p$ and energy $E < -m$ corresponds to emission of an antiparticle with 4-momentum $p$ and positive energy $E > m$.

Since $T_{ik}(s, t)$ is a function of kinematical invariants (not on the sign of $P_i$), the same function describes the following reactions:

1. $1+2 \rightarrow 3+4$ for $P_1, P_2, P_3, P_4 > 0$ $\quad s$ – channel ($s > 4m^2$, $t, u < 0$)
2. $1+3 \rightarrow 2+4$ for $P_1, P_4 > 0$ and $P_2, P_3 < 0$ $\quad t$ – channel ($t > 4m^2$, $s, u < 0$)
3. $1+4 \rightarrow 2+3$ for $P_1, P_3 > 0$ and $P_2, P_4 < 0$ $\quad u$ – channel ($u > 4m^2$, $s, t < 0$)
4. $1 \rightarrow 2+3+4$ for unstable particle ($P_1, P_3, P_4 > 0$ and $P_2 < 0$)
Unitarity

From the conservation of the probability norm in interaction processes:

\[ S^+ S = 1 \]

The sum of probabilities of all processes which are possible at a given energy is equal to unity

\[
i \left[ T_{fk}^* - T_{fk} \right] = \sum_n \int \prod_{l=1}^{n} \frac{d^3 p_l}{2E_l(2\pi)^3} T_{nk} T_{fn}^* \left( \frac{2\pi}{\hbar} \right)^4 \delta^{(4)}(P_k - \sum_{q=1}^{n} P_q)
\]

\( T_{nk} \) – the amplitude for a transition from the state \(| k \rangle\) to the state \(| n \rangle\) with \( n \) particles

\[
\sum_n \int \quad \text{means integration over phase-space for each particle in the channel with} \ n \ \text{particles sum over all channels.}
\]

Optical theorem

If \(| f \rangle = | k \rangle\):

\[
2 \text{Im} T(s, 0) = \sum_n \int \prod_{l=1}^{n} \frac{d^3 p_l}{2E_l(2\pi)^3} |T_{nk}|^2 \left( \frac{2\pi}{\hbar} \right)^4 \delta^{(4)}(P_k - \sum_{q=1}^{n} P_q) = 4j\sigma_{tot}(s)
\]

\[
\frac{d\sigma_{el}}{dt} = \frac{|T(s,t)|^2}{64\pi p_a^2 s} \quad \sigma_{tot}(s) = \frac{\text{Im} T(s,0)}{2p_a \sqrt{s}}
\]

\[
j = \sqrt{(p_a p_b)^2 - m_a m_b^2}
\]
Kinematics of binary reactions

Let's assume $m_1 = m_2 = m_3 = m_4 = m$

In CM system: $p_1 + p_2 = p_3 + p_4 = 0$

$$E_i = \frac{1}{2} \sqrt{s} \quad p_i^2 = \frac{s}{4} - m^2 \quad i = 1, 2, 3, 4$$

$$t = -(p_1 - p_3)^2 = -2p_1^2(1 - \cos\theta_s)$$

$$-4p_1^2 \leq t \leq 0$$

$$\cos\theta_s = 1 + \frac{2t}{s - 4m^2} = -\left(1 + \frac{u}{s - 4m^2}\right)$$

$$T(s,t) = T(s, \cos\theta_s)$$

$$\frac{d\sigma_{el}}{d\Omega} = \left|\frac{T(s,t)}{8\pi\sqrt{s}}\right|^2$$
**Partial wave expansion**

\[
f(s, \cos \theta) = \frac{1}{8\pi \sqrt{s}} T(s, t) = \frac{1}{P} \sum_{l=0}^{\infty} (2l+1) f_l(s) P_l(\cos \theta)
\]

Represented in the form of a series in the partial-wave amplitudes \(f_l(t)\), which characterize scattering in the state with relative orbital angular momentum \(l\).

\[
\int_{-1}^{1} dz P_l(z) P_{l'}(z) = \frac{2 \delta_{ll'}}{2l+1} + \text{unitarity}
\]

\[
P_l(z) = \frac{1}{2^l l!} \left( \frac{dz}{dz} \right)^l (z^2 - 1)^l - \text{Legendre polynomial}
\]

\[
\text{Im } f_l(s) = \left| f_l(s) \right|^2 + \sum_n \int \left| f_N^N(s, \tau_N) \right|^2 d\tau_N \quad \Rightarrow \quad \text{Im } f_l(s) \geq \left| f_l(s) \right|^2 \quad \Rightarrow \quad \text{Im } f_l(s) \leq 1
\]

**Froissart bound**

For \(s \to \infty\) the contribution comes from terms with \(l_{\text{eff}} \leq C \sqrt{s} \ln s\)

\[
\sigma_{\text{tot}}(s) = \frac{\text{Im } T(s, 0)}{2p_*^\sqrt{s}} = \frac{4\pi}{P_a^2} \sum_{l=0}^{l_{\text{eff}}} (2l+1) f_l(s) \leq \frac{4\pi}{P_a^2} l_{\text{eff}}^2 \approx C' \ln^2 s
\]

\[
\sigma_{\text{tot}}(s) \leq C' \ln^2 s
\]

(assumes unitarity, analyticity, short-range character of strong interactions)
Pomeranchuk theorem

\[ \text{Im} T_{ab}(s, t = 0) = s \sigma_{tot}(ab) \]
\[ \text{Im} T_{ab}(u, t = 0) = u \sigma_{tot}(\bar{a}b) \]

If \( T \) is not an oscillating function and \( \frac{1}{\ln s} \frac{\text{Re} T(s, 0)}{\text{Im} T(s, 0)} \to 0 \) at \( s \to \infty \)

\[ \sigma_{tot}(ab) = \sigma_{tot}(\bar{a}b) \text{ at } s \to \infty \]

Pomeranchuk theorem may be violated. See O. Nachtmann’s talk.
Suppose the $f_i(t)$ has a singularity of form

$$f_i(t) = \frac{r(t)}{l - \alpha(t)}$$

$$\frac{1}{2i}
\left[ f_i(t) - f_i^*(t) \right] \sim f_i(t)f_i^*(t)$$

$$f_i(t) - f_i^*(t) = \frac{r}{l - \alpha(t)} - \frac{r^*}{l - \alpha^*(t)} = \frac{r\left(l - \alpha^*(t)\right) - r^*\left(l - \alpha(t)\right)}{(l - \alpha(t))(l - \alpha^*(t))} \sim 2i \frac{rr^*}{(l - \alpha(t))(l - \alpha^*(t))}$$

$$r = r^*$$

No poles in the real axis in the $l$ plane for $t > 4m^2$
Resonances

Assume for some \( t = t_R \equiv M_R^2 > 4m^2 \), \( \text{Re} \alpha(t_R) = l_R \) → \( \alpha(t) \approx l_R + i \text{Im} \alpha(t_R) + \alpha'(t_R)(t - t_R) \)

Taylor expansion

\[
f_{l_R}(t) \approx \frac{r(t_1)}{l_R - [l_R + i \text{Im} \alpha(t_R) + \alpha'(t_R)(t - t_R)]} = - \frac{\text{Im} \alpha(t_1)}{\alpha'(t_R)(t - t_R) + i \text{Im} \alpha(t_R)} = \frac{\text{Im} \alpha(t_R) / \alpha'(t_R)}{t_R - t - i \text{Im} \alpha(t_R) / \alpha'(t_R)}
\]

\( \text{Im} \alpha(t_R) / \alpha'(t_R) \equiv M_R \Gamma \)

Breit-Wigner

\[
f_{l_R}(t) \sim \frac{1}{M_R^2 + t - iM_R \Gamma}
\]

Regge pole in the physical region of the \( t \)-channel \( (t > 4m^2) \) corresponds to a Breit-Wigner resonance with \( \text{Re} \alpha(t_1) = l_R \) (=spin of \( R \))
Reggeon trajectory

Regge trajectories are almost straight lines and in standard Regge theory they are parameterized by $\alpha(t) = \alpha_0 + \alpha' \cdot t$

Regge pole gives a generalization of a particle exchange in the $t$-channel. It corresponds to an exchange in the $t$-channel by a state of noninteger spin $\alpha(t)$ (reggeon trajectory), which coincides with particles of spin $J$ for $t = M_J^2$
Partial wave expansion in t-channel and Sommerfeld-Watson transformation

\[ T(t,s) = T(t,\cos\theta_t) = \sum_{l=0}^{\infty} (2l + 1) f_l(t) P_l(\cos\theta_t) \]

\[ \cos\theta_t = 1 + \frac{2s}{t - 4m^2} \]

\[ \sin(\pi\alpha) \approx \sin(\pi l) + \pi(\alpha - l)\cos(\pi l) = \pi(-1)^l(\alpha - l) \]

\[ (2l + 1)f_l(t)P_l(\cos\theta_t) = \frac{1}{2i} \int_{C_i} \frac{(-1)^\alpha(2\alpha + 1)f(\alpha, t)P_\alpha(\cos\theta_t)}{\sin(\pi\alpha)} d\alpha \]

\[ T(t,s) = \frac{1}{2i} \int_{C} e^{-i\pi\alpha}(2\alpha + 1)f(\alpha, t)P_\alpha(\cos\theta_t) \frac{d\alpha}{\sin(\pi\alpha)} \]

\[ T(t,s) = \frac{1}{2i} \int_{C} \sum_{\sigma = \pm 1} \left( 1 + \sigma e^{-i\pi\alpha} \right)(2\alpha + 1)f^{\sigma}(\alpha, t)P_\alpha(\cos\theta_t) \frac{d\alpha}{2\sin(\pi\alpha)} \]

\[ T(t,s) = \sum_{\sigma = \pm 1 \text{ poles}} \sum_{\sigma} \eta_\sigma(\alpha) \left( \alpha_i^{\sigma}(t) \right) r_i^{\sigma}(t) P_{\alpha_i^{\sigma}(t)}(\cos\theta_t) \]

\[ \eta_\sigma(\alpha) = -\frac{\sigma + e^{-i\pi\alpha}}{\sin(\pi\alpha)} \quad \text{signature factor} \]

Cauchy’s integral theorem

\[ F(a) = \frac{1}{2\pi i} \int_{C} \frac{F(z)}{z - a} dz \]
Regge pole exchange amplitude

“Physical” region in the $t$–channel corresponds to $t > 4m^2$, $s < 0$. Analytically continue the amplitude to $s > 4m^2$, $t < 0$ ($s$–channel).

For $s >> 4m^2 > |t|$, $\cos \theta_t \sim \frac{s}{4m^2} >> 1$

$$P_i(z) \sim z^l \quad \text{for } z >> 1$$

$$T(t,s) = \sum_{\sigma = \pm 1} \sum_{\text{poles}} \eta_{\sigma} \left( \alpha_i^\sigma(t) \right) \gamma_i^\sigma(t) \left( \frac{s}{s_0} \right)^{\alpha_i^\sigma(t)}$$

$$\sigma_{tot}(s) = \frac{1}{s} \text{Im}T(s,0) \sim s^{\alpha_0-1}$$

$s_0$ is a constant scale factor, usually chosen to be $s_0 = 1 \text{ GeV}^2$.
Duality

\[ \text{res.} = \sum \text{resonances} \]

\[ = \sum \text{reggeons} \]
Factorization

What is the meaning of $\gamma(t)$?

In fact, all information about incoming and outgoing particles (baryon number, strangeness, etc.) are absorbed in $\gamma(t)$ and it does not depend on $s$.

$\gamma(t)$ should be related to Reggeon-hadron interaction vertex!

One can assume the initial state does not know anything about the final state: in the cross-channel the initial particles first transform into an intermediate state, which then gets converted into the final particles, with the amplitude independent of the properties of the initial state.

$$\gamma(t) = g_{aa}(t)g_{bb}(t)$$

It is not possible to predict the explicit form of $g_{aa}(t)$ from the analytical properties of the matrix element (model dependent).
Regge pole approximation

At fixed $t$, with $s >> t$

- Amplitude for a process governed by the exchange of a trajectory $\alpha(t)$ is
  \[ T(s,t) \propto \left( \frac{s}{s_0} \right)^{\alpha(t)} \]

- No prediction for $t$ dependence

- Elastic cross section
  \[ \frac{d\sigma_{el}}{dt} \approx \frac{1}{s^2} |T(s,t)|^2 \propto s^{2(\alpha(t)-1)} \]

- Total cross section considering the optical theorem
  \[ \sigma_{tot} \approx \frac{1}{s} \text{Im} T(s,0) \propto s^{\alpha(0)-1} \]
Reggeons

\[ \alpha_i(t) = \alpha_i(0) + \alpha_i' \cdot t, \quad i = f, \rho, \omega. \]

\[
\begin{align*}
\alpha_f(0) &= 0.703 \pm 0.023 & \alpha_f'(0) &= 0.797 \pm 0.014 \text{GeV}^{-2} \\
\alpha_\rho(0) &= 0.522 \pm 0.009 & \alpha_\rho'(0) &= 0.809 \pm 0.015 \text{GeV}^{-2} \\
\alpha_\omega(0) &= 0.435 \pm 0.033 & \alpha_\omega'(0) &= 0.923 \pm 0.054 \text{GeV}^{-2}
\end{align*}
\]

\[
\sigma_{tot} \propto \left( \frac{S}{S_0} \right)^{\alpha_i(0)-1} \quad \rightarrow \quad \sigma_{tot} \rightarrow 0 \quad \text{at} \quad s \rightarrow \infty
\]
Experiment: \( \sigma_{tot} \rightarrow 0 \) at \( s \rightarrow \infty \)

\[ \sigma_{tot} \sim \left( \frac{s}{s_0} \right)^{\alpha_i(0)-1} \]

An object with \( \alpha(0) = 1 + \Delta > 1 \) is needed
Donnachie and Landshoff (1992)

\[ \sigma_{\text{tot}} = A s^{0.0808} \]

grows as a power function of \( s \)

Unitarity requires that the total cross section at very high energies should not grow faster than \( \ln^2 s \) (Froissart bound).

For describing DIS data

\[ F_2(x, Q^2) = f(Q^2) x^{\Delta(Q^2)} \]

(CKMT 1992)

\[ F_2(x, Q^2) = f_1(Q^2) x^{-0.08} + f_2(Q^2) x^{-0.42} \]

(DL 1998)

soft Pomeron hard Pomeron
It is usually assumed that the Pomeron in QCD is related to gluonic exchanges in the t–channel.

$\Delta_{\text{eff}}$ determined from fits to data are in general different from $\Delta$.

DIFFRACTION:
In HEP any process involving Pomeron exchange

See talks by
L. Jenkovszky
A. Martin
O. Nachtmann
W. Schäfer
A simple parameterization of Regge residues

\[ g_{aa}(t) = g_{aa} \exp\left\{ R_{aa}^2 t \right\} \]

\[ R_{aa} \text{ -- Regge radius of hadron } a \]

\[ \alpha(t) = \alpha_0 + \alpha' t \]

\[ \eta_\sigma(\alpha) = -\frac{\sigma + e^{-i\pi\alpha}}{\sin(\pi\alpha)} = e^{-i\pi\alpha/2} \]

\[ \lambda \equiv R_{aa}^2 + R_{bb}^2 + \alpha' \left( \ln\left(\frac{s}{s_0}\right) - i\pi/2 \right) \]

Impact parameter representation

\[ f_{ab}(s, b) \sim \int d^2 q_\perp \exp\left\{ -ibq_\perp \right\} T(s, q_\perp^2) \sim \frac{(s/s_0)^{\alpha_0 - 1}}{\lambda} \exp\left\{ -\frac{b^2}{4\lambda} \right\} \]

\[ \frac{d\sigma}{dt} \sim \left( \frac{s}{s_0} \right)^{2\alpha - 2} \times \exp\left\{ -2 \left( R_{aa}^2 + R_{bb}^2 + \alpha' \ln\left(\frac{s}{s_0}\right) \right) |t| \right\} \]

\[ \sqrt{b^2} = 2 |\lambda| \approx 2 \sqrt{R_{aa}^2 + R_{bb}^2 + \alpha' \ln\left(\frac{s}{s_0}\right)} \]

Radius of interaction increases with increasing \( s \)

Increases with increasing \( s \). Diffraction peak shrinkage.
Unitarity and two-body & three-body reactions

\[ \sum_c T_{ac} T_{ac}^* = 2 \Im m T_{aa} \]

Analogous to the optical theorem, Muller’s theorem relates the inclusive cross-section for the reaction \( h_1 + h_2 \rightarrow c + X \) to the forward scattering amplitude of the three-body hadronic process \( h_1 + h_2 + \bar{c} \rightarrow h_1 + h_2 + \bar{c} \).

Double Regge limit

\[ \Rightarrow 2 \Im m \]

Triple Regge limit

\[ \Rightarrow 2 \Im m \]
$\frac{d\sigma_{SD}}{dM^2 dt} = \left(\frac{s_0}{s}\right)^2 \sum_{i,j,k} G_{ijk}(t) \left(\frac{s}{M^2}\right)^{\alpha_i(t)+\alpha_j(t)} \left(\frac{M^2}{s_0}\right)^{\alpha_k(0)}$

$G_{ijk}(t) = 4\pi g_{aa}^{\alpha_i}(t) g_{aa}^{\alpha_j}(t) g_{bb}^{\alpha_k}(0) r_{\alpha_i\alpha_j}(t) \eta(\alpha_i(t)) \eta^*(\alpha_j(t))$

See talks by
A. Martin
L. Jenkovszky

8/25/13 Martin Poghosyan
Double diffraction

Central production

Double gap topology
**s-channel picture of Reggeons**

Multiperipheral fluctuation development time:

\[ \tau = \frac{p}{m^2} \]

Slow partons interact: \( p_n \approx m \)

\[ n \sim \ln p \sim \ln s \]

Random walk in \( b \) space: \( \sqrt{b^2} \sim n \sim \sqrt{\ln s} \)

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High-energy hadronic interactions are essentially non-local.
Summation of multiperipheral diagrams leads to regge behavior

Reggeon is a non-local object!

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Ladder diagram
See A. Martin’s talk
Space-time picture of high-energy hh interactions

AFS (successive)

Mandelstam (simultaneous)

\[\tau \sim E\]

\[\sigma \to 0 \quad at \quad s \to \infty\]
Regge poles in QCD

$1/N$-expansion is a useful non-perturbative method to study soft interaction dynamics.

$N_c >> 1$ (t’Hooft)

$N_c \approx N_f >> 1$ (Veneziano)

All diagrams are classified according to their topology. Amplitudes are expanded in $1/N$ ($1/N^2$). The first term corresponds to planar diagrams.

Cutting of the planar diagram in the $s$-channel

Configuration of the final state particles.
**Pomeron in QCD**

Pomeron is usually related to gluonic exchange in the $t$-channel. From the point of view of $1/N$-expansion Pomeron corresponds to cylinder-type diagrams.

Cutting of the cylindrical diagram in the $s$-channel

Configuration of the final state particles.
\[ a = \pi^+ \begin{cases} \frac{u}{d} \end{cases} \]

\[ b = \pi^- \begin{cases} \frac{d}{u} \end{cases} \]

\[ \sigma_{ab}^{an}(y_a - y_b) = w(y_{qa} - y_{\bar{q}a})w(y_{qb} - y_{\bar{q}b}) \cdot \sigma_{\bar{q}aqb} \cdot P_{qaqb \to X} \]

\[ w(y_{qa} - y_{\bar{q}a})w(y_{qb} - y_{\bar{q}b}) = Aw(y_a - y_b) \]

\[ w(y_{qi} - y_{\bar{q}j}) = A \exp\left\{-\beta(y_{qi} - y_{\bar{q}j})\right\} \]

\[ \int d^2b_q w(y_a - y_{\bar{q}}, b_a - b_{\bar{q}})w(y_b - y_q, b_q - b_b) = Aw(y_a - y_b, b_a - b_b) \]

\[ w(y_i - y_k, b_i - b_k) = \frac{A}{4\pi\gamma(y_i - y_k)} \exp\left\{-\beta(y_i - y_k) - \frac{(b_i - b_k)^2}{4\gamma(y_i - y_k)}\right\} \]

\[ \beta = 1 - \alpha_R(0), \quad \gamma = \alpha'_R \]
At high energies (parton densities) the interaction between Pomerons starts to play an important role. The Regge theory becomes unsafe. Interaction vertices (multi-Pomeron and Pomeron-hadron) are not known theoretically.

**models based on RFT:**
Kaidalov-Ponomarev-Ter-Martirosyan, Khoze-Martin-Ryskin, Gotsman-Levin-Maor, Ostapchenko, L. Jenkovszky et al., Kaidalov-Poghosyan, ...

Main difference in implementing the GW mechanism, in used sets of diagrams, and in parameterizing interaction vertices (+AGK).
Gribov’s reggeon calculus

Regge-poles are not the only singularities of the amplitude. There are also branch points which correspond to the exchange of several Reggeons. A Regge pole can be interpreted as corresponding to a single scattering. Regge cuts – multiple scatterings of hardons’ constituents.

\[ iM^{(n)}(s, t) = \frac{1}{n!} \int \prod_{i=1}^{n} \left[ iM^{(1)}(s, q_{i\perp}^2) \frac{d^2q_{i\perp}}{\pi} \right] C^{(n)}(\{q_{i\perp}\}) \delta \left( q_{\perp} - \sum_{i=1}^{n} q_{i\perp} \right) \]

\[ M^{(1)}(s, t) = \frac{T(s,t)}{8\pi s} = \gamma \eta(\alpha(0)) e^{\lambda t} \left( \frac{s}{s_0} \right)^\Delta \]

\[ \Delta = \alpha_p - 1 \]
Multi-Pomeron exchange

\[ M_P^{(n)}(s,t) = -i\lambda \left( \frac{\gamma}{\lambda} \right)^n \exp\left\{ -\frac{\lambda t}{n} \right\} \frac{n!}{n \cdot n!} \left( -\frac{s}{s_0} \right)^{n\Delta} \]

\[ \Delta_{nP} = n\Delta_P \rightarrow \text{for } \Delta_P > 0 \text{ all } nP \text{ exchanges should be taken into account} \]

\[ M(s,t) = \sum_{n=1}^{\infty} M_P^{(n)}(s,t) \]

\[ \sigma_{\text{tot}} \sim \ln^2 s \]

Impact parameter representation

\[ F(s,b) = 1 - \exp\left[ \chi_P(s,b) \right] \]

\[ \chi_P(s,b) = -\frac{\gamma}{\lambda} \exp\left\{ -\Delta \ln \left( \frac{s}{s_0} \right) - b^2/4\lambda \right\} \]

- eikonal

\[ 2\sqrt{\alpha_P/\Delta} \ln \left( \frac{s}{s_0} \right) \]

exponential (as for single \( P \) exchange)
How to calculate the cross-section of a given process?
Abramovsky-Gribov-Kancheli cutting rules

AGK cutting rules allow:
• to relate to each other the different s-channel discontinuities of a given graph
• to calculate the contribution of each graph in the total cross-section.

• If the Pomeron is not cut entirely, its contribution is suppressed exponentially.
• No particle production from interaction vertices
• All the vertices for various cuts are the same and real.

  o There is one cut-plane which separates the initial and final states

  o Each cut-pomeron obtains an extra factor of (-2) due to the discontinuity of the pomeron amplitude (for a cut Pomeron replace the factor $iM^{(1)}(s,t)$ by $2\text{Im}M^{(1)}(s,t)$ )

  o Each un-cut pomeron obtains an extra factor of 2 since it can be placed on both sides of the cut-plane (the factors $iM^{(1)}(s,t)$ for the Pomerons to the right of the cut are placed by the complex-conjugate values)
AGK for PP exchange

\[
2\Delta M_0^{(2)} = 2 \left[ \text{Re}M^{(1)}(s,t_1)\text{Re}M^{(1)}(s,t_2) + \text{Im}M^{(1)}(s,t_1)\text{Im}M^{(1)}(s,t_2) \right]
\]

Diffractive cutting (between Pomerons)

\[
2\Delta M_1^{(2)} = -8 \left[ \text{Im}M^{(1)}(s,t_1)\text{Im}M^{(1)}(s,t_2) \right]
\]

Cutting through one of Pomerons

\[
2\Delta M_2^{(2)} = 4 \left[ \text{Im}M^{(1)}(s,t_1)\text{Im}M^{(1)}(s,t_2) \right]
\]

Cutting both Pomerons

\[
2\Delta M_0^{(2)} + 2\Delta M_1^{(2)} + 2\Delta M_2^{(2)} = 2 \left[ \text{Re}M^{(1)}(s,t_1)\text{Re}M^{(1)}(s,t_2) - \text{Im}M^{(1)}(s,t_1)\text{Im}M^{(1)}(s,t_2) \right] = 2\Delta M^{(2)}
\]

\[
\Delta M_1^{(2)} + 2 \cdot \Delta M_2^{(2)} = 0
\]
For any $n \geq k > 0$

$$2\Delta M_k^{(n)}(s, t) = (-1)^{n-k}2^n C_n^k \prod_{i=1}^n \text{Im}M^{(1)}(s, t_i)$$

$$2 \sum_{k=1}^n k\Delta M_k^{(n)}(s, t) = \left[ (-2)^n \prod_{i=1}^n \text{Im}M^{(1)}(s, t_i) \right] \sum_{k=1}^n (-1)^k k C_n^k = 0 \quad n \geq 2$$
AGK and multiparticle production

\[ N_{\text{part}}^*(-4) + 2N_{\text{part}}^*(+2) = 0 \]  

(AGK cancellation)

\( \sigma_n^{(2)} \) is negative, it is a correction to the pole diagram. Reducing it opens a room for new production processes.
Inclusive cross-section

The central part of the inclusive spectrum is determined by Mueller-Kancheli diagram:

\[
\left( \frac{d\sigma}{dy} \right)_{y=0} \sim s^{(\alpha_{R_1} + \alpha_{R_2} - 2)/2}
\]

if \( R_1 = R_2 \equiv P \):

\[
\left( \frac{d\sigma}{dy} \right)_{y=0} \sim s^\Delta
\]

With account of enhanced diagrams only Mueller-Kancheli type diagrams survive

Schwimmer model:

\[
\left( \frac{d\sigma}{dy} \right)_{y=0} \propto \frac{s^\Delta}{1 + \frac{g_p r_{3P}}{\Delta} (s^\Delta - 1)}
\]
First estimate of the influence of enhanced graphs on physical observables

Dubovikov et al., Nucl. Phys. B123
Kopelovich and Lapidus, Sov. Phys. JETP 44
Dubovikov and Ter-Martirosyan, Nucl. Phys. B124
Kaidalov et al., Sov. J. N.P. 44

\[ \Delta_{\text{eff}} = \Delta - 4\pi G_{\text{PPP}} \]

\[ \Delta \approx 0.2 \quad \Delta_{\text{eff}} \approx 0.12 \]
First estimate of the influence of enhanced graphs on physical observables

$\Delta \approx 0.2 \quad \Delta_{\text{eff}} \approx 0.12$

Kaidalov et al., Sov. J. N.P. 44

Dubovikov et al., Nucl. Phys. B123

Kopelovich and Lapidus, Sov. Phys. JETP 44

Kaidalov et al., Sov. Phys. JETP 44

Inel

NSD
**KNO-scaling violation was predicted**

Number of chains increases with energy ➔ no KNO scaling at high energies

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Kaidalov and Ter-Martirosyan 1982

\[ \psi(z) = \langle N_{ch} \rangle \sigma_{N_{ch}}/\sigma_{\text{in}} \]

\[ \sqrt{s} = 10^6 \text{ GeV} \]

\[ \sqrt{s} = 540 \text{ GeV} \]
The role of multiple rescattering in hard processes

Survival probability (Bjorken 1992): no other interactions occur except the hard coll. of interest

$$S^2 = \frac{\int |M(s,b)|^2 P(s,b) d^2b}{\int |M(s,b)|^2 d^2b}$$

- $M(s,b)$ - amplitude (in $b$-space) of the particular process
- $P(s,b)$ - probability that no inelastic interaction occurs between scattered hadrons

Interplay of “soft” and “hard” dynamics in QCD.

Strong suppression of inelastic diffraction in the region of small $b$ ($P \rightarrow 0$). Inelastic diff. occurs at the periphery of interaction region, where nonperturbative effects are essential.
At $E < E_c$ ($E_c \sim m_R$) an elastic $hA$ -scattering amplitude can be considered as successive rescatterings of an initial hadron on nucleons of a nucleus. (Glauber)

At high energies hadronic (nuclear) fluctuations are “prepared” long before the interaction.

For $E > E_c$ there is a coherent interaction of constituents of a hadron with nucleons of a nucleus. $hA$ elastic amplitude can be calculated as in the Glauber model, but with account of diffractive intermediate states. (Gribov)

**The role of enhanced diagrams in AA**

For collisions of identical nuclei (SS, PbPb) the $A^{4/3}$-dependence of particle densities of eq. (10) typical for the Glauber model changes to the behaviour $A^\delta$. The value of delta is a weak function of energy and it is equal to $\delta \approx 1.1$ at LHC energies.

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*Heavy Ion Phys. 9 (1999, published before the RHIC era!)*

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$dN/d\eta$ depends on $A^{4/3}$ or $A^{1.1}$?

**$A^{4/3}$ - Gribov-Glauber without enhanced diagrams**

- 19.6 GeV: AuAu PHOBOS
- 22.4 GeV: CuCu PHOBOS
- 62.4 GeV: AuAu PHOBOS
- 130 GeV: AuAu PHENIX, AuAu PHOBOS

**$A^{1.1}$ - Gribov-Glauber with enhanced diagrams**

- 19.6 GeV: AuAu PHENIX, AuAu PHOBOS
- 22.4 GeV: CuCu PHOBOS
- 62.4 GeV: AuAu PHOBOS, AuAu PHENIX
- 200 GeV: AuAu BRAHMS
- 130 GeV: AuAu PHENIX, AuAu PHOBOS

$dN_{ch}/d\eta \propto A^{4/3} \cdot s^{0.15}$

$dN_{ch}/d\eta \propto A^{1.1} \cdot s^{0.15}$

Centrality (%)
Books and review papers
